B

Register Number | | | | | | |

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. If $\vec{PR} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{QS} = -\vec{i} + 3\vec{j} + 2\vec{k}$ then the area of the quadrilateral *PQRS* is

   a) $5\sqrt{3}$ b) $10\sqrt{3}$

   c) $5\frac{\sqrt{3}}{2}$ d) $\frac{3}{2}$.

2. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j} + 2\vec{k}$ then a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is

   a) $\frac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}$ b) $\frac{\vec{i} - \vec{j} + \vec{k}}{\sqrt{3}}$

   c) $\frac{-\vec{i} + \vec{j} + 2\vec{k}}{\sqrt{3}}$ d) $\frac{\vec{i} - \vec{j} - \vec{k}}{\sqrt{3}}$

[Turn over

3. The point of intersection of the lines

$\vec{r} = \left(-\vec{i} + 2\vec{j} + 3\vec{k}\right) + t\left(-2\vec{i} + \vec{j} + \vec{k}\right)$ and

$\vec{r} = \left(2\vec{i} + 3\vec{j} + 5\vec{k}\right) + s\left(\vec{i} + 2\vec{j} + 3\vec{k}\right)$ is

a) (2, 1, 1)
b) (1, 2, 1)
c) (1, 1, 2)
d) (1, 1, 1).

4. The distance from the origin to the plane $\vec{r} \cdot \left(2\vec{i} - \vec{j} + 5\vec{k}\right) = 7$ is

a) $\frac{7}{\sqrt{30}}$
b) $\frac{\sqrt{30}}{7}$
c) $\frac{30}{7}$
d) $\frac{7}{30}$.

5. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular unit vectors then

$\left|\vec{a} + \vec{b} + \vec{c}\right|$ is

a) 3
b) 9
c) $3\sqrt{3}$
d) $\sqrt{3}$.

6. If the line $5x - 2y + 4k = 0$ is a tangent to $4x^2 - y^2 = 36$, then k is

a) $\frac{4}{9}$
b) $\frac{2}{3}$
c) $\frac{9}{4}$
d) $\frac{81}{16}$.

7. The eccentricity of the hyperbola with asymptotes $x + 2y - 5 = 0$,

$2x - y + 5 = 0$ is

a) 3
b) $\sqrt{2}$
c) $\sqrt{3}$
d) 2.

8. The point of contact of the parabola $y^2 = 4ax$ and the tangent $y = mx + c$ is

a) $\left(\frac{2a}{m^2}, \frac{a}{m} \right)$

b) $\left(\frac{a}{m^2}, \frac{2a}{m} \right)$

c) $\left(\frac{a}{m}, \frac{2a}{m^2} \right)$

d) $\left(-\frac{a}{m^2}, -\frac{2a}{m} \right)$.

9. The slope of the normal to the curve $y = 3x^2$ at the point whose x-coordinate = 2, is

a) $\frac{1}{13}$

b) $\frac{1}{14}$

c) $-\frac{1}{12}$

d) $\frac{1}{12}$.

10. If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero, is

a) $\frac{32}{3}$ m/sec

b) $-\frac{16}{3}$ m/sec

c) $\frac{16}{3}$ m/sec

d) $-\frac{32}{3}$ m/sec.

11. If p is true and q is false then which of the following is not true ?

a) $p \to q$ is false

b) $p \vee q$ is true

c) $p \wedge q$ is false

d) $p \leftrightarrow q$ is true.

12. Given $E(X+C) = 8$ and $E(X-C) = 12$ then the value of C is

a) -2

b) 4

c) -4

d) 2.

13. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is

a) $\frac{5}{3}$

b) $\frac{3}{5}$

c) $\frac{5}{9}$

d) $\frac{9}{5}$.

14. If $f(x)$ is a *p.d.f.* of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x)\,dx$ is

a) 1

b) 0·5

c) 0

d) 0·25.

15. If X is a discrete random variable then $P(X \geq a)$ is equal to

a) $P(X < a)$

b) $1 - P(X \leq a)$

c) $1 - P(X < a)$

d) 0.

16. If $A = [\,2 \;\; 0 \;\; 1\,]$ then rank of AA^T is

a) 1 b) 2

c) 3 d) 0.

17. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & k & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has inverse then

a) k is any real number

b) $k = -4$

c) $k \neq -4$

d) $k \neq 4$.

18. If A is a matrix of order 3, then det (kA)

a) k^3 det (A)

b) k^2 det (A)

c) k det (A)

d) det (A).

19. Every homogeneous system

a) is always consistent

b) has only trivial solution

c) has infinitely many solutions

d) need not be consistent.

20. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a}+\vec{b}\right)$ is a unit vector if

a) $\theta = \frac{\pi}{3}$

b) $\theta = \frac{\pi}{4}$

c) $\theta = \frac{\pi}{2}$

d) $\theta = \frac{2\pi}{3}$.

21. A particular integral of $\left(D^2 - 4D + 4\right) y = e^{2x}$ is

a) $\frac{x^2}{2} e^{2x}$

b) xe^{2x}

c) xe^{-2x}

d) $\frac{x}{2} e^{-2x}$.

22. The order and degree of the differential equation $y' + \left(y''\right)^2 = x\left(x + y''\right)^2$ respectively are

a) 1, 1

b) 2, 1

c) 2, 2

d) 1, 2.

23. Which of the following is a contradiction ?

a) $p \vee q$

b) $p \wedge q$

c) $p \vee (\sim p)$

d) $p \wedge (\sim p)$.

24. The order of [7] in $\left(Z_9 , +_9 \right)$ is

a) 9

b) 6

c) 3

d) 1.

25. Which of the following is correct ?

a) An element of a group can have more than one inverse

b) If every element of a group is its own inverse, then the group is Abelian

c) The set of all 2×2 real matrices forms a group under matrix multiplication

d) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$.

26. The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x-axis, is

a) 18π

b) 2π

c) 36π

d) 9π.

27. The arc length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

a) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$

b) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx$

c) $\int_c^d \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dx$

d) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dx}{dy} \right)^2} \, dx$.

28. The surface area of the solid of revolution of the region bounded by $y = 2x$, $x = 0$ and $x = 2$ about x-axis, is

a) $8\sqrt{5}\,\pi$

b) $2\sqrt{5}\,\pi$

c) $\sqrt{5}\,\pi$

d) $4\sqrt{5}\,\pi$.

29. The differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is

a) $\frac{d^2y}{dx^2} + ay = 0$

b) $\frac{d^2y}{dx^2} - 9y = 0$

c) $\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0$

d) $\frac{d^2y}{dx^2} + 9x = 0$.

30. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x^2}$ is

a) e^x

b) $\log x$

c) $\frac{1}{x}$

d) e^{-x}.

31. Which of the following functions is increasing in $(0, \infty)$?

a) e^x

b) $\frac{1}{x}$

c) $-x^2$

d) x^{-2}.

32. One of the critical numbers of $x^{3/5}(4-x)$ is

a) 4 b) $\frac{4}{3}$

c) 0 d) $\frac{3}{5}$.

33. The percentage error in the 11^{th} root of the number 28 is approximately .... times the percentage error in 28.

a) $\frac{1}{28}$ b) $\frac{1}{11}$

c) 11 d) 28.

34. The curve $9y^2 = x^2(4-x^2)$ is symmetrical about

a) y-axis only

b) x-axis only

c) $y = x$

d) both the axes.

35. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

a) $\frac{3\pi}{16}$

b) $\frac{3}{16}$

c) 0

d) $\frac{3\pi}{8}$.

36. The polar form of the complex number $\left(i^{25}\right)^3$ is

a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

b) $\cos\pi + i\sin\pi$

c) $\cos\pi - i\sin\pi$

d) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$.

37. If $z_1 = 4 + 5i$ and $z_2 = -3 + 2i$ then $\frac{z_1}{z_2}$ is

a) $\frac{2}{13} - \frac{22}{13}i$

b) $\frac{-2}{13} + \frac{22}{13}i$

c) $\frac{-2}{13} - \frac{23}{13}i$

d) $\frac{2}{13} + \frac{22}{13}i$.

38. The value of $\sqrt{z\bar{z}}$ is

a) $|z|^2$

b) $|z|$

c) $2|z|$

d) $2|z|^2$.

39. If $a = 3 + i$ and $z = 2 - 3i$ then the points on the Argand diagram representing az, $3az$ and $-az$ are

a) vertices of a right-angled triangle

b) vertices of an equilateral triangle

c) vertices of an isosceles triangle

d) collinear.

40. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is

a) $\sqrt{7}$

b) 4

c) 3

d) 5.

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

42. Solve the following non-homogeneous system of linear equations by determinant method :

$$4x + 5y = 9, \quad 8x + 10y = 18.$$

43. a) Show that the vectors $3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\vec{i} + \vec{j} - 4\vec{k}$ form a right-angled triangle.

b) A force of magnitude 5 units acting parallel to $2\vec{i} - 2\vec{j} + \vec{k}$ displaces the point of application from (1, 2, 3) to (5, 3, 7). Find the work done.

44. Show that $\left[\vec{a} - \vec{b}\,,\ \vec{b} - \vec{c}\,,\ \vec{c} - \vec{a} \right] = 0$.

45. Find the square root of $(-8 - 6i)$.

46. The headlight of a motor vehicle is a parabolic reflector of diameter 12 cm and depth 4 cm. Find the position of the bulb on the axis of the reflector for effective functioning of the headlight.

47. a) Using Rolle's theorem find the value of c for $f(x) = \sqrt{1-x^2}$, $-1 \leq x \leq 1$.

b) Obtain the Maclaurin's series for e^x.

48. Evaluate : $\lim_{x \to 1} x^{\frac{1}{x-1}}$.

49. Given $w = \dfrac{x}{x^2 + y^2}$ where $x = \cos t$; $y = \sin t$. Find $\dfrac{dw}{dt}$.

50. Evaluate : $\displaystyle\int_0^{2\pi} \sin^9 \frac{x}{4}\, dx$.

51. Solve : $\dfrac{dy}{dx} + \dfrac{4x}{x^2+1}\, y = \dfrac{1}{\left(x^2+1\right)^2}$.

52. Construct the truth table for $\sim [(\sim p) \wedge (\sim q)]$.

53. Show that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

54. In a Poisson distribution if $P(X=2) = P(X=3)$, find $P(X=5)$.

[Given $e^{-3} = 0{\cdot}050$].

55. a) Find the mean and variance for the probability density function

$$f(x) = \begin{cases} \dfrac{1}{24}, & -12 \leq x \leq 12 \\ 0, & \text{otherwise} \end{cases}$$

OR

b) Simplify : $\dfrac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$.

SECTION – C

N. B. : i) Answer *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Solve by matrix inversion method :

$$2x - y + 3z = 9, \quad x + y + z = 6, \quad x - y + z = 2.$$

57. Prove by vector method : Altitudes of a triangle are concurrent.

58. Find the vector and Cartesian equations of the plane through the points (1, 2, 3) and (2, 3, 1) and perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

59. Find the axis, vertex, focus, equation of the directrix, equation of the latus rectum and length of the latus rectum of the parabola $y^2 + 8x - 6y + 1 = 0$ and hence sketch its graph.

60. An arch is in the form of a semi-ellipse whose span is 48 feet wide. The height of the arch is 20 feet. How wide is the arch at a height of 10 feet above the base ?

61. Find the equation of the rectangular hyperbola which has one of its asymptotes $x + 2y - 5 = 0$ and passes through the points (6, 0) and (– 3, 0).

62. Show that the equation of the normal to the curve $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$ at θ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.

63. Trace the curve $y = x^3$.

64. Find the area between the curve $y = x^2 - x - 2$, x-axis and the lines $x = -2$, $x = 4$.

65. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r (b - a)$ and hence deduce the surface area of the sphere $(b > a)$.

66. Solve : $\left(D^2 - 2D + 2 \right) y = \sin 2x + 5$.

67. The sum of Rs. 1,000 is compounded continuously at the nominal rate of interest 4 per cent per annum. In how many years will the amount be twice the original principal ? $\left(\log_e 2 = 0{\cdot}6931 \right)$.

68. Prove that the set of four functions f_1, f_2, f_3, f_4 on the set of non-zero complex numbers $\mathbb{C} - \{ 0 \}$ defined by $f_1 (z) = z$, $f_2 (z) = -z$, $f_3 (z) = \frac{1}{z}$, $f_4 (z) = -\frac{1}{z}$, $\forall z \in \mathbb{C} - \{ 0 \}$ forms an Abelian group with respect to composition of functions.

69. A random variable X has the following probability mass function :

x	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

i) Find k

ii) Evaluate $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X \leq 6)$

iii) Find the smallest value of x for which $P(X \leq x) > \frac{1}{2}$.

70. a) If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that

$\alpha^n - \beta^n = i\, 2^{n+1} \sin \frac{n\pi}{3}$ and deduce $\alpha^9 - \beta^9$.

OR

b) A poster is to have an area of 180 cm^2 with 1 cm margins at the bottom and sides and a 2 cm margin on the top. What dimensions will give the largest printed area ?