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Register Number

Part III — MATHEMATICS

(New Syllabus)

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. :   i)   *All* questions are compulsory.

ii)   Each question carries *one* mark.

iii)   Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1.   ' $\div$ ' is a binary operation on

   a) N

   b) R

   c) Z

   d) $\subset - \{ 0 \}$.

2.   If $f(x) = \dfrac{A}{\pi} \cdot \dfrac{1}{16 + x^2}$, $-\infty < x < \infty$ is a *p.d.f.* of a continuous random variable X, then the value of A is

   a)   16

   b)   8

   c)   4

   d)   1.

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3. *Var* (4 *X* + 3) is

   a)   7

   b)   16 *Var* (*X*)

   c)   19

   d)   0.

4. In a Poisson distribution, if $P (X = 2) = P (X = 3)$, then the value of its parameter λ is

   a)   6

   b)   2

   c)   3

   d)   0.

5. For the *p.d.f.* of the normal distribution function

   $$f(x) = C\, e^{- x^2 + 3x}; \quad -\infty < x < \infty,\ \text{the mean}\ \mu\ \text{is}$$

   a)   3

   b) $\dfrac{2}{3}$

   c) $\dfrac{3}{2}$

   d)   6.

6. The area between the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and its auxiliary circle ($a > b$) is

   a) $\pi b (a - b)$

   b) $2\pi a (a - b)$

   c) $\pi a (a - b)$

   d) $2\pi b (a - b)$.

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7. The volume, when the curve $y = \sqrt{3 + x^2}$ from $x = 0$ to $x = 4$ is **rotated** about x-axis, is

    a) $100\,\pi$

    b) $\dfrac{100}{9}\,\pi$

    c) $\dfrac{100}{3}\,\pi$

    d) $\dfrac{100}{3}$.

8. The area of the curve $y^2 = (x - 5)^2 (x - 6)$ between $x = 5$ and $x = 6$ is

    a) 0   b) 1

    c) 4   d) 6.

9. The differential equation $\left(\dfrac{dx}{dy} \right)^2 + 5y^{1/3} = x$ is

    a) of order 2 and degree 1

    b) of order 1 and degree 2

    c) of order 1 and degree 6

    d) of order 1 and degree 3.

10. The integrating factor of the differential equation $\dfrac{dy}{dx} + Py = Q$ is

    a) $\displaystyle\int P\,dx$ b) $\displaystyle\int Q\,dx$

    c) $e^{\int Q\,dx}$ d) $e^{\int P\,dx}$.

11. The sum of the distances of any point on the ellipse $4x^2 + 9y^2 = 36$ from $\left(\sqrt{5},\ 0 \right)$ and $\left(-\sqrt{5},\ 0 \right)$ is

    a) 4   b) 8

    c) 6   d) 18.

12. The directrix of the hyperbola $x^2 - 4(y-3)^2 = 16$ is

    a) $y = \pm \dfrac{8}{\sqrt{5}}$

    b) $x = \pm \dfrac{8}{\sqrt{5}}$

    c) $y = \pm \dfrac{\sqrt{5}}{8}$

    d) $x = \pm \dfrac{\sqrt{5}}{8}$.

13. The equations of transverse and conjugate axes of the hyperbola

    $144x^2 - 25y^2 = 3600$ respectively are

    a) $y = 0$; $x = 0$

    b) $x = 12$; $y = 5$

    c) $x = 0$; $y = 0$

    d) $x = 5$; $y = 12$.

14. The slope of the normal to the curve $y = 3x^2$ at the point whose x-coordinate is

    2, is

    a) $\dfrac{1}{13}$

    b) $\dfrac{1}{14}$

    c) $-\dfrac{1}{12}$

    d) $\dfrac{1}{12}$.

15. The value of C in Rolle's Theorem for the function $f(x) = \cos \dfrac{x}{2}$ on $[\pi, 3\pi]$

    is

    a) 0

    b) 2π

    c) $\dfrac{\pi}{2}$

    d) $\dfrac{3\pi}{2}$.

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16. If $\left[\vec{a} \times \vec{b} , \ \vec{b} \times \vec{c} , \ \vec{c} \times \vec{a} \right] = 64$ then $\left[\vec{a} , \ \vec{b} , \ \vec{c} \right]$ is

   a)   32   b)   8

   c)   128   d)   0.

17. The work done by the force $\vec{F} = \vec{i} + \vec{j} + \vec{k}$ acting on a particle, if the particle is displaced from A (3, 3, 3) to the point B (4, 4, 4), is

   a)   2 units   b)   3 units

   c)   4 units   d)   7 units.

18. The shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \ \text{ and } \ \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} \ \text{ is}$$

   a) $\dfrac{2}{\sqrt{3}}$

   b) $\dfrac{1}{\sqrt{6}}$

   c) $\dfrac{2}{3}$

   d) $\dfrac{1}{2\sqrt{6}}$.

19. The angle between the vectors $\vec{i} - \vec{j}$ and $\vec{j} - \vec{k}$ is

   a) $\dfrac{\pi}{3}$ b) $-\dfrac{2\pi}{3}$

   c) $-\dfrac{\pi}{3}$ d) $\dfrac{2\pi}{3}$.

20. The centre and radius of the sphere $\left| \vec{r} - \left(2\vec{i} - \vec{j} + 4\vec{k} \right) \right| = 5$ are

   a)   (2, – 1, 4) and 5

   b)   (2, 1, 4) and 5

   c)   (– 2, 1, 4) and 6

   d)   (2, 1, – 4) and 5.

21. The particular integral of $\left(3D^2 + D - 14 \right) y = 13e^{2x}$ is

    a) $26\, xe^{2x}$

    b) $13\, xe^{2x}$

    c) xe^{2x}

    d) $\dfrac{x^2}{2}\, e^{2x}$.

22. The differential equation corresponding to $y = ax^2 + bx + c$ where $\{ a, b, c \}$ are arbitrary constants, is

    a) $y''' + y'' = 0$

    b) $y'' = 2a$

    c) $y''' = 0$

    d) $y''' - y'' = 0$.

23. The conditional statement $p \rightarrow q$ is equivalent to

    a) $p \lor q$

    b) $p \lor (\sim q)$

    c) $(\sim p) \lor q$

    d) $p \land q$.

24. Which of the following is a contradiction ?

    a) $p \lor q$

    b) $p \land q$

    c) $p \lor (\sim p)$

    d) $p \land (\sim p)$.

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25. The value of $[3] +_{11} ([5] +_{11} [6])$ is

   a) $[0]$

   b) $[1]$

   c) $[2]$

   d) $[3]$.

26. In a given semicircle of diameter 4 cm a rectangle is to be inscribed. The area of the largest rectangle is

   a) 2

   b) 4

   c) 8

   d) 16.

27. The condition for the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ to cut orthogonally is that

   a) $\dfrac{1}{a} + \dfrac{1}{c} = \dfrac{1}{b} + \dfrac{1}{d}$

   b) $\dfrac{1}{a} - \dfrac{1}{c} = \dfrac{1}{b} - \dfrac{1}{d}$

   c) $\dfrac{1}{a} + \dfrac{1}{c} = \dfrac{1}{b} - \dfrac{1}{d}$

   d) $\dfrac{1}{a} - \dfrac{1}{c} = \dfrac{1}{b} + \dfrac{1}{d}$.

28. The percentage error in the 11th root of the number 28 is approximately .... times the percentage error in 28.

   a) $\dfrac{1}{28}$

   b) $\dfrac{1}{11}$

   c) 11

   d) 28.

29. The curve $9y^2 = x^2 \left(4 - x^2 \right)$ is symmetrical about

   a) y-axis only

   b) x-axis only

   c) $y = x$

   d)   both the axes.

30. The value of $\int_0^1 x \left(1 - x \right)^4 \, dx$ is

   a) $\dfrac{1}{12}$ b) $\dfrac{1}{30}$

   c) $\dfrac{1}{24}$ d) $\dfrac{1}{20}$.

31. The polar form of the complex number $\left(i^{25} \right)^3$ is

   a) $\cos \dfrac{\pi}{2} + i \sin \dfrac{\pi}{2}$

   b) $\cos \pi + i \sin \pi$

   c) $\cos \pi - i \sin \pi$

   d) $\cos \dfrac{\pi}{2} - i \sin \dfrac{\pi}{2}$.

32. The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

   a) i

   b) $- i$

   c) 1

   d) $- 1$.

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33. If ω is a cube root of unity then the value of $\left(1 - \omega + \omega^2 \right)^4 + \left(1 + \omega - \omega^2 \right)^4$ is

a)   0

b)   32

c) -16

d) -32.

34. If $a + ib = (8 - 6i) - (2i - 7)$ then the values of a and b respectively are

a)   8, -15

b)   8, 15

c)   1, 4

d)   15, -8.

35. The line $2x + 3y + 9 = 0$ touches the parabola $y^2 = 8x$ at the point

a) $(0, -3)$

b) $(2, 4)$

c) $\left(-6, \dfrac{9}{2} \right)$

d) $\left(\dfrac{9}{2}, -6 \right)$.

36. If A is a square matrix of order n, then $| adj\, A |$ is

a) $| A |^2$

b) $| A |^n$

c) $| A |^{n-1}$

d) $| A |$.

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37. The inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is

a) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$

d) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$.

38. If the equations $-2x + y + z = l,\ x - 2y + z = m,\ x + y - 2z = n,$ such that $l + m + n = 0,$ then the system has

a) a non-zero unique solution

b) trivial solution

c) infinitely many solutions

d) no solution.

39. $\left(A^T \right)^{-1}$ is equal to

a) A^{-1}

b) A^T

c) A

d) $\left(A^{-1} \right)^T.$

40. If $|\ \vec{a} + \vec{b}\ | = |\ \vec{a} - \vec{b}\ |$ then

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $|\ \vec{a}\ | = |\ \vec{b}\ |$

d) $\vec{a}$ and $\vec{b}$ are unit vectors.

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SECTION – B

N. B. : i)   Answer any *ten* questions.

ii)   Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii)   Each question carries *six* marks.   10 × 6 = 60

41. For $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$, show that $A = A^{-1}$.

42. a)   For any vector $\vec{r}$, prove that

$$\vec{r} = (\vec{r} \cdot \vec{i})\,\vec{i} + (\vec{r} \cdot \vec{j})\,\vec{j} + (\vec{r} \cdot \vec{k})\,\vec{k}$$

    b)   If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 60$, then find $|\vec{a} \times \vec{b}|$

43. Forces $2\vec{i} + 7\vec{j}$, $2\vec{i} - 5\vec{j} + 6\vec{k}$, $-\vec{i} + 2\vec{j} - \vec{k}$ act at a point P whose position vector is $4\vec{i} - 3\vec{j} - 2\vec{k}$. Find the moment of the resultant of three forces acting at P about the point Q whose position vector is $6\vec{i} + \vec{j} - 3\vec{k}$.

44. Find the square roots of $-8 - 6i$.

45. P represents the variable complex number z. Find the locus of P

if $|2z - 1| = |z - 2|$.

46. The headlight of a motor vehicle is a parabola reflector of diameter 12 cm and depth 4 cm. Find the position of bulb on the axis of the reflector for effective functioning of the headlight.

47. Obtain the Maclaurin's series for $\log_e (1 + x)$.

48. Find the intervals in which $f(x) = x^3 - 3x + 1$ is increasing or decreasing.

49. If $u = (x - y)(y - z)(z - x)$ then show that $u_x + u_y + u_z = 0$.

50. Evaluate : $\displaystyle\int_0^3 \frac{\sqrt{x}\, dx}{\sqrt{x} + \sqrt{3 - x}}$.

51. Solve : $\dfrac{dy}{dx} + y \cot x = 2 \cos x$.

52. Show that $\sim (p \lor q) \equiv (\sim p) \land (\sim q)$.

53. Find the probability distribution of the number of sixes in throwing three dice once.

54. Find the mean and variance of the distribution with *p.d.f.*

$$f(x) = \begin{cases} xe^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

55. a) Find the rank of $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$.

OR

b) State and prove cancellation laws on groups.

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SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Show that the equations $x + y + z = 6$, $x + 2y + 3z = 14$ and $x + 4y + 7z = 30$ are consistent and solve them by using rank.

57. Prove that $\cos(A + B) = \cos A \cos B - \sin A \sin B$ by vector method.

58. Find the vector and Cartesian equations of the plane through the points $(1, 2, 3)$, $(2, 3, 1)$ and perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

59. If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot \theta = y + 1$, show that

$$\frac{(y + \alpha)^n - (y + \beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}.$$

60. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

61. Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$, $(2, 4)$ is the centre of the hyperbola and it passes through $(2, 0)$.

62. A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at the rate of $2 \ m^3$ /min, find the rate at which the water level is rising when the water is 3 m deep.

63. Find the absolute and local maximum and minimum values of

$$f(x) = 1 - x^2 ; \ -2 \le x \le 1.$$

64. Trace the curve $y^2 = 2x^3$.

65. Find the common area enclosed by the parabolas $4y^2 = 9x$ and $3x^2 = 16y$.

66. Find the volume of the solid generated by the revolution of the loop of the curve $x = t^2 ; \ y = t - \dfrac{t^3}{3}$ about x-axis.

67. Solve : $\left(D^2 - 6D + 9 \right) y = x + e^{2x}$.

68. Show that the set G of all positive rationals forms a group under the composition $*$ defined by $a * b = \dfrac{ab}{3}$ for all $a, b \in G$.

69. The mean height of 500 male students in a certain college is 151 pounds and the standard deviation is 15 pounds. Assuming the weights are normally distributed, find how many students weigh

   i) between 120 and 155 pounds

   ii) more than 185 pounds.

   $\big[\ P [\ 0 < z < 2 \cdot 067] = 0 \cdot 4803 ; \ P [\ 0 < z < 0 \cdot 2667] = 0 \cdot 1026 ;$

   $P [\ 0 < z < 2 \cdot 2667] = 0 \cdot 4881 \big]$.

70. a) Find the eccentricity, centre, foci and vertices of the ellipse

$36x^2 + 4y^2 - 72x + 32y - 44 = 0$ and also trace the curve.

OR

b) Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years ?

$\left[\text{Take } A_0 \text{ as the initial amount}\right]$.