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Register Number

Part III — MATHEMATICS

(New Syllabus)

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. The particular integral of the differential equation $f(D)_y = e^{ax}$ where $f(D) = (D-a)\,g(D)$, $g(a) \neq 0$, is

a) me^{ax}

b) $\dfrac{e^{ax}}{g(a)}$

c) $g(a)\,e^{ax}$

d) $\dfrac{xe^{ax}}{g(a)}$.

2. The order and degree of the differential equation

$\sin x\,(dx + dy) = \cos x\,(dx - dy)$ are

a) 1, 1 b) 0, 0

c) 1, 2 d) 2, 1.

[Turn over

3. The number of rows in the truth table of $\sim [p \wedge (\sim q)]$ is

a) 2 b) 4

c) 6 d) 8.

4. In a set of integers with operation $*$ defined by $a * b = a + b - ab$, the value of $3 * (4 * 5)$ is

a) 25 b) 15

c) 10 d) 5.

5. In the multiplicative group of n^{th} roots of unity, the inverse of ω^k, where $k < n$, is

a) $\omega^{\frac{1}{k}}$

b) ω^{-1}

c) ω^{n-k}

d) $\omega^{\frac{n}{k}}$.

6. The curve $y = ax^3 + bx^2 + cx + d$ has a point of inflexion at $x = 1$, then

a) $a + b = 0$ b) $a + 3b = 0$

c) $3a + b = 0$ d) $3a + b = 1$.

7. $\lim_{x \to 0} \frac{x}{\tan x}$ is

a) 1 b) -1

c) 0 d) ∞.

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8. If $u = \log\left[\frac{x^2+y^2}{xy}\right]$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

a) 0 b) u

c) $2u$ d) u^{-1}.

9. An asymptote to the curve $y^2(a+2x) = x^2(3a-x)$ is

a) $x = 3a$ b) $x = -\frac{a}{2}$

c) $x = \frac{a}{2}$ d) $x = 0$.

10. The area of the region bounded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$ is

a) $\sqrt{2}+1$ b) $\sqrt{2}-1$

c) $2\sqrt{2}-2$ d) $2\sqrt{2}+2$.

11. The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

a) i b) $-i$

c) 1 d) -1.

12. If p represents the variable complex number z and if $|2z-1| = 2|z|$, then the locus of p is

a) the straight line $x = \frac{1}{4}$

b) the straight line $y = \frac{1}{4}$

c) the straight line $z = \frac{1}{2}$

d) the circle $x^2 + y^2 - 4x - 1 = 0$.

13. If ω is a cube root of unity, then the value of

$$\left(1 - \omega + \omega^2\right)^4 + \left(1 + \omega - \omega^2\right)^4 \text{ is}$$

a) 0

b) 32

c) -16

d) -32.

14. The arguments of n^{th} roots of a complex number differ by

a) $\frac{2\pi}{n}$

b) $\frac{\pi}{n}$

c) $\frac{3\pi}{n}$

d) $\frac{4\pi}{n}$.

15. If B and B' are the ends of the minor axis, F_1 and F_2 are the foci of the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, then the area of $F_1 BF_2 B'$ is

a) 16

b) 8

c) $16\sqrt{2}$

d) $32\sqrt{2}$.

16. If A is a square matrix of order n, then $|\ adj\,(A)\ |$ is

a) $|\ A\ |^2$

b) $|\ A\ |^n$

c) $|\ A\ |^{n-1}$

d) $|\ A\ |$.

17. In a system of 3 linear non-homogeneous equations with three unknowns, if $\Delta = 0$ and $\Delta_x = 0$, $\Delta_y \neq 0$ and $\Delta_z = 0$, then the system has

a) unique solution

b) two solutions

c) infinitely many solutions

d) no solution.

18. If the equations $-2x + y + z = l$, $x - 2y + z = m$ and $x + y - 2z = n$ are such that $l + m + n = 0$, then the system has

a) a non-zero unique solution

b) trivial solution

c) infinitely many solutions

d) no solution.

19. If $\rho(A) = \rho(A, B)$ = the number of unknowns, then the system is

a) consistent and has infinitely many solutions

b) consistent and has unique solution

c) consistent

d) inconsistent.

20. If $\vec{u} = \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b})$, then

a) $\vec{u}$ is a unit vector

b) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$

c) $\vec{u} = \vec{0}$

d) $\vec{u} \neq \vec{0}$.

21. ' + ' is not a binary operation on

a) N

b) Z

c) $\mathbb{C}$

d) $Q - \{0\}$.

22. $Var(4x + 3)$ is

a) 7

b) $16\ Var(X)$

c) 19

d) 0.

23. For a binomial distribution with mean 2 and variance $\frac{4}{3}$, p is equal to

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) $\frac{3}{4}$

d) $\frac{2}{\sqrt{3}}$.

24. The random variable X follows a normal distribution whose probability function is given by $f(x) = ce^{\frac{-\frac{1}{2}(x-100)^2}{25}}$. The value of c is

a) $\sqrt{2\pi}$

b) $\frac{1}{\sqrt{2\pi}}$

c) $5\sqrt{2\pi}$

d) $\frac{1}{5\sqrt{2\pi}}$.

25. In a Poisson distribution if $P[X = 2] = P[X = 3]$, then the value of its parameter λ is

a) 6 b) 2

c) 3 d) 0.

26. The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle is $(a > b)$

a) $\pi b(a - b)$

b) $2\pi a(a - b)$

c) $\pi a(a - b)$

d) $2\pi b(a - b)$.

27. $\int_0^\infty x^5 e^{-4x} dx$ is

a) $\frac{\lfloor 6}{4^6}$

b) $\frac{\lfloor 6}{4^5}$

c) $\frac{\lfloor 5}{4^6}$

d) $\frac{\lfloor 5}{4^5}$.

28. The volume of the solids obtained by revolving the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about its major and minor axes are in the ratio $(a > b)$

a) $b^2 : a^2$ b) $a^2 : b^2$

c) $a : b$ d) $b : a$.

29. If $y = ke^{\lambda x}$ then its differential equation is

a) $\frac{dy}{dx} = \lambda y$

b) $\frac{dy}{dx} = ky$

c) $\frac{dy}{dx} + ky = 0$

d) $\frac{dy}{dx} = e^{\lambda x}$.

30. On putting $y = vx$ the homogeneous differential equation

$x^2 dy + y(x + y) dx = 0$ becomes

a) $x dv + (2v + v^2) dx = 0$

b) $v dx + (2x + x^2) dv = 0$

c) $v^2 dx - (x + x^2) dv = 0$

d) $v dv + (2x + x^2) dx = 0$.

31. The eccentricity of the hyperbola whose latus rectum is equal to half of its conjugate axis, is

a) $\frac{\sqrt{3}}{2}$

b) $\frac{5}{3}$

c) $\frac{3}{2}$

d) $\frac{\sqrt{5}}{2}$.

32. If P is any point on the hyperbola $\frac{x^2}{36} - \frac{y^2}{4} = 1$ and the ordinate at P meets the asymptotes at Q and Q' then $QP \cdot Q'P$ is

a) 36

b) 6

c) 4

') 2.

33. The equations of the major and minor axes of $\frac{x^2}{9} + \frac{y^2}{4} = 1$ respectively are

a) $x = 3, \; y = 2$

b) $x = -3, \; y = -2$

c) $x = 0, \; y = 0$

d) $y = 0, \; x = 0$.

34. The surface area of a sphere, when the volume is increasing at the same rate as its radius, is

a) 1

b) $\frac{1}{2\pi}$

c) 4π

d) $\frac{4\pi}{3}$.

35. The angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is

a) $2 \tan^{-1}\left(\frac{3}{4}\right)$

b) $\tan^{-1}\left(\frac{4}{3}\right)$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{4}$.

36. If $\left| \vec{a} + \vec{b} \right| = \left| \vec{a} - \vec{b} \right|$, then

a) $\vec{a}$ is parallel to $\vec{b}$

b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $\left| \vec{a} \right| = \left| \vec{b} \right|$

d) $\vec{a}$ and $\vec{b}$ are unit vectors.

37. The projection of $\vec{OP}$ on a unit vector $\vec{OQ}$ equals thrice the area of parallelogram $OPRQ$. Then $\angle POQ$ is

a) $\tan^{-1}\left(\frac{1}{3}\right)$

b) $\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)$

c) $\sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$

d) $\sin^{-1}\left(\frac{1}{3}\right)$.

38. The two lines $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z-1}{2}$ are

a) parallel

b) intersecting

c) skew

d) perpendicular.

39. The projection of $\vec{i} - \vec{j}$ on Z-axis is

a) 0

b) 1

c) -1

d) ∞.

40. The unit normal vectors to the plane $2x - y + 2z = 5$ are

a) $2\vec{i} - \vec{j} + 2\vec{k}$

b) $\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$

c) $-\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$

d) $\pm\frac{1}{3}\left(2\vec{i} - \vec{j} + 2\vec{k}\right)$.

SECTION - B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. If $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

42. Solve the following system of linear equation by determinant method :

$$2x + 3y = 8, \quad 4x + 6y = 16.$$

43. Show that the points $(3, -1, -1)$, $(1, 0, -1)$ and $(5, -2, -1)$ are collinear.

44. Find the vector and Cartesian equations of a sphere with centre having position vector $2\vec{i} - \vec{j} + 3\vec{k}$ and radius 4 units.

45. Solve $x^4 + 4 = 0$, if $1 + i$ is one of the roots.

46. Prove that the tangent at any point to the rectangular hyperbola forms with the asymptotes a triangle of constant area.

47. Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1, \quad -\frac{1}{2} \le x \le 4.$$

48. Verify Lagrange's law of mean for the function $f(x) = x^3 - 5x^2 - 3x$ on $[1, 3]$

49. Find an approximate value for $\sqrt[3]{65}$ by using differentials.

50. Find the area of the region bounded by $y = 2x + 4$, $y = 1$, $y = 3$ and y-axis.

51. Solve $x^2 \frac{dy}{dx} = y^2 + 2xy$ given that $y = 1$ when $x = 1$.

52. a) Construct the truth table for $(p \vee q) \wedge (\sim q)$.

b) Show that $p \wedge (\sim p)$ is a contradiction.

53. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

54. a) The difference between the mean and the variance of a binomial distribution is 1 and the difference between their squares is 11. Find n.

b) Show that the total probability is 1 (for a Poisson distribution).

55. a) Marks in an aptitude test given to 800 students of a school was found to be normally distributed. 10% of the students scored below 40 marks and 10% of the students scored above 90 marks. Find the number of students scored between 40 and 90 marks.

OR

b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$ and $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Discuss the solutions of the system of equations for all values of λ (use rank method) :

$$x + y + z = 2, \quad 2x + y - 2z = 2, \quad \lambda x + y + 4z = 2.$$

57. Find the vector and Cartesian equations of the plane passing through the point (– 1, – 2, 1) and perpendicular to two planes $x + 2y + 4z + 7 = 0$ and $2x - y + 3z + 3 = 0$.

58. Solve the equation : $x^9 + x^5 - x^4 - 1 = 0$.

59. Find the eccentricity, centre, foci, vertices of the ellipse

$36x^2 + 4y^2 - 72x + 32y - 44 = 0$ and sketch the graph.

60. A cable of a suspension bridge is in the form of a parabola whose span is 40 m. The roadway is 5 m below the lowest point of the cable. An extra support is provided across the cable 30 m above the ground level. Find the length of the support if the heights of the pillars are 55 m.

[Turn over

61. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere).

62. Find the intervals of concavity and the points of inflexion of the function $f(x) = x^4 - 6x^2$.

63. Verify Euler's theorem for $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$.

64. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\cot x}}$.

65. Find the perimeter of the circle with radius a, by using integration.

66. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the bacteria triple in 1 hour, show that the number of bacteria at the end of five hours will be 3^5 times of the population at initial time.

67. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$ when $x = \log 2$, $y = 0$ and when $x = 0$, $y = 0$.

68. Show that the set G of all positive rationals forms a group under the composition $*$, defined by $a * b = \frac{ab}{3}$ for all $a, b \in G$.

69. The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equals to 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accident in a year, (ii) more than 3 accidents in a year $[e^{-3} = 0.0498]$.

70 a) Find the equation of the hyperbola if its asymptotes are parallel to $x + 2y - 12 = 0$ and $x - 2y + 8 = 0$ respectively, (2, 4) is the centre of the hyperbola and the hyperbola passes through (2, 0).

OR

b) Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x-2}{1} = \frac{y-1}{2} = \frac{-z-1}{1}$ intersect and find the point of intersection.